

# Experiences of using Evolutionary Techniques in Logic Minimisation

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## Abstract:

This paper reviews our work in recent years applying Evolutionary Techniques to the minimisation of logic functions. This work has primarily concerned AND-EXOR (Reed-Muller Logic) networks, as distinct from the more traditional Boolean form (AND-OR). We have also applied these techniques to the minimisation of the logic required to realise sequential state machines (State Assignment Problem). Recently we have been more specifically targeting these minimised functions for Field Programmable Gate arrays (FPGAs).

## 1. Overview of Reed-Muller Logic.

Traditionally digital electronic logic circuits have been implemented using Boolean devices such as Inclusive-OR gates, AND gates, NOT gates etc. However it has been known for some time that it is possible to implement logic functions using an alternative form of logic known as Reed-Muller logic [Green86]. In this form of logic the Inclusive-OR is replaced by the Exclusive-OR operation. This relatively unexplored form of logic implementation allows a much greater number of possible representations of a target logic function than in the Boolean case. On many occasions circuits implemented in Reed-Muller logic are more compact than their Boolean counterparts [Sasao93] and, as a consequence integrated circuits employing Reed-Muller logic may require a smaller area of silicon and so be cheaper to produce, or alternatively it may mean a given function can reside on a smaller FPGA part. In addition many Reed-Muller implementations are known to be easier to test [Reddy72, Chatterjee95]. The testability of ICs is now an extremely important commercial consideration.

The techniques developed for optimisation of circuit complexity (in particular the number of gates required to implement the function) in the Boolean domain cannot be applied to the Reed-Muller form. Consequently there has been much research in recent years in developing techniques for optimisation of gate requirements in the Reed-Muller field [see Sasao95]. However so large are the number of possible alternative but equivalent representations that researchers have largely concentrated on very specialised forms known as fixed-polarity expansions. Initially our own work was in this field. However fixed-polarity representations are not appropriate for industrial benchmark logic functions with very large numbers of inputs and outputs. Consequently we have recently concentrated on the optimisation of logic functions as a so-called Exclusive-OR sum-of-products(ESOP). This is the most general representation of a Reed-Muller logic function. Other researchers recently [see Sasao95] have attempted to optimise representations in this domain with varying degrees of success.

## 2. Genetic Algorithms and Stochastic Search Techniques applied to Logic Minimisation.

There is growing interest in the use of Genetic Algorithms (GAs) [Holland75, Goldberg89] and associated evolutionary techniques to the problem encountered in logic optimisation. Initially workers studied the fixed polarity problem in Reed-Muller Logic [Miller94b, Becker94, Zeng95]. An  $n$ -variable Boolean function has  $2^n$  possible Reed-Muller expansions in the fixed polarity form, some of which may be considerably simpler than others. Early work concentrated on heuristic approaches [McKenzie93]. It turned out that these techniques would often terminate on a local sub-optimum when there still existed better solutions elsewhere in the search space. We showed that these drawbacks were more pronounced as the size of the problem increased and we found that a genetic algorithm was capable of producing good quality solutions beyond the range of the heuristic methods [Miller94b, Miller94c]. We used a very simple chromosome representation in that we merely encoded the polarity as a binary string, and used traditional one-point crossover. The fitness criterion was the number of EX-OR gates in the candidate solution. We also found unusually small population sizes to be very effective. Unlike the heuristic techniques, which are deterministic, the GA offers the potential for further quality results when it is run a number of times, or left to run longer.

In an attempt to assess the comparative merits of different search strategies for the fixed polarity problem we looked at the performance of :

- GA with two different parameter sets,
- Evolutionary Strategy [Back91,Rechenberg73],
- Tabu Search [Glover89, Glover90]
- Stochastic Nearest Ascent Hillclimber [Reeves93]
- Stochastic Steepest Ascent Hillclimber [Reeves93]

We found many of these techniques to give good results as detailed in [Robertson96]. It is interesting to note that these particular problems were amenable to local search techniques and we feel that a combination of GA and local search might prove to be even more effective.

The encouraging results obtained in [Miller94b] led us to try similar techniques on the more general Reed-Muller expansions namely the Exclusive-OR sum-of products (ESOP) form. This type of representation of a Boolean function allows variables to appear complemented or uncomplemented without restriction, consequently the search space is vastly expanded, however more compact representations become more likely. An attempt to formulate a simple representation of the ESOP expansion in the form of a chromosome was based on the following idea. A gene value of one corresponding to a particular literal  $x_i$  could encode for that literal to be expanded as  $1+!x_i$  where the plus indicates EX-OR and ! indicates complementation, and a gene value of zero would indicate no expansion. This representation suffered from two particular drawbacks. Firstly the number of genes was equal to the number of literals in the initial specification, giving rise to potentially very large chromosomes. Secondly the number of new terms created by multiplication of expanded literals could become enormous leading to prohibitive evaluation time and memory requirements.

It was clear for the above reasons that we had to find a formulation of the ESOP minimisation process which would allow chromosome sizes to remain manageable. We decided to view the problem in the framework of a binary decision diagram [Akers78] with fixed decision variables throughout each level of the tree. Such a representation of the problem allowed us then to encode a chromosome as corresponding to a permutation of input variables. We developed symbolic simplification rules operating in

a ‘bottom-up’ fashion, rather than the more usual top-down method. The result we obtained was a multi-level symbolic representation of the initial function which required considerably fewer 2-input gates than the original circuit [Thomson96]. Results obtained using industry standard benchmark files in .pla format showed this new method to produce circuits of considerably less complexity than either fixed polarity representations (having been exhaustively searched [Miller94a]) or 2-level ESOPs [Sasao93]. We encoded the chromosome representation for the decision variable ordering in the form of an ‘ordinal list’ [Michalewicz92].

One shortcoming of [Thomson96] was that the algorithm required the initial circuit specification to be presented in minterm form, so that .pla files which contained product terms with variables missing had to be expanded. This obviously limited the algorithm to smaller problems. In [Miller95] the algorithm was generalised in that a ternary representation was used, this removed the need to expand product terms into a completely specified form. The total number of unique, mutually exclusive algebraic simplification rules derived from a four node ternary sub-tree proved to be extremely large. This led us to develop an alternative simplification strategy, with a manageable number of rules. However we found that the simplification rules were non-commutative under certain circumstances and that the rule sequence could affect the amount of simplification possible and so lead to differences in circuit complexity. To address this difficulty we developed a double chromosome GA in which the first chromosome encoded for variable ordering in the decision tree and the second chromosome encoded for the sequence in which the simplification rules were applied. This produced superior results on the same benchmarks than the single chromosome version with fixed rule ordering.

In [Miller96] various combinations of Tabu Search with a GA were investigated for the multi-level ternary simplification. It was found that some form of embedded local search was advantageous.

Another area of logic optimisation which has been investigated using GAs has been the so-called state-assignment problem in the synthesis of sequential state machines. This is again a combinatorial problem with an enormous search space. We have found a GA to be very effective for this problem and we obtained favourable results when compared with well-known heuristics [Almaini95].

One of the obstacles inherent in these areas is the amount of computation required to evaluate the fitness of a candidate solution. In all of our experiments we have tended to use very small populations, and limit the number of unique evaluations performed in a given run. We have found a look-up table, containing already evaluated solutions, was very useful in preventing expensive re-evaluation of individuals, especially near convergence.

### **3. Conclusions and Further Work.**

We have surveyed our recent work in the field of logic minimisation, where we have found evolutionary techniques to be extremely useful. Recently we have compared the configurable logic block (CLB) requirements on Xilinx FPGAs for implementations of industrial .pla benchmarks using misII the well known multi-level Boolean minimisation package against the same benchmarks in AND-EXOR multi-level form. We have found many cases where our algorithms produced more compact circuits with lower CLB counts than misII. In addition many of our designs were easier to route, this is very encouraging.

We are currently considering modifications to chromosome fitness calculation to take into account fan-out and timing constraints. Such modifications are relatively easy to implement in our evolutionary algorithms due to the fact that chromosome fitness is based on the *actual* implementation cost.

#### 4. References

- [Akers78] S. B. Akers, "Binary Decision Diagrams", *IEEE Trans. Comput.*, Vol. C-27, No. 6, pp. 509-516, 1978.
- [Almaini95] A.E.A. Almaini, J.F. Miller, P. Thomson, S. Billina, "State assignment of finite state machines using a genetic algorithm", *IEE Proc. Comput. Digit. Tech.* Vol. 142, No. 4, pp. 279-286, 1995
- [Back91] T. Back, F Hoffmeister, H-P. Schwefel, "A survey of evolutionary strategies", *Proc. ICGA IV, Morgan Kaufmann 1991*.
- [Becker94] B. Becker, R. Dreschler, "OFDD based Minimisation of Fixed Polarity Reed-Muller Expressions Using Hybrid Genetic Algorithms", *Proc IEEE Int. Conf. on Computer Design, October 1994*.
- [Chatterjee95] M. Chatterjee, D. K. Pradhan, and W. Kunz, "LOT: Logic Optimization with Testability - New Transformations using Recursive Learning", *ICCAD, 1995*.
- [Glover89] F.Glover, "Tabu search - part I", *ORSA J. Comput*, Vol. 1, No. 3, pp. 190-206, 1989.
- [Glover90] F.Glover, "Tabu search - part II", *ORSA J. Comput*, Vol. 2, No. 1, pp. 4-32, 1990.
- [Goldberg89] D.E. Goldberg, "Genetic Algorithms in Search, Optimisation and Machine Learning", *Addison-Wesley, 1989*.
- [Green86] D. Green, "Modern Logic Design", *Addison-Wesley, 1986*
- [Holland75] J.H. Holland, "Adaptation in Natural and Artificial Systems", *University of Michigan Press, 1975*.
- [McKenzie93] L. McKenzie, A.E.A. Almaini, J. F. Miller, P. Thomson. "Optimisation of Reed-Muller logic functions", *Int. J. Electronics*, Vol.75, No. 3, pp. 451-466, 1993
- [Michalewicz92] Z. Michalewicz, "Genetic Algorithms + Data Structures = Evolution Programs.", *Springer-Verlag 1992*.
- [Miller94a] J.F.Miller and P.Thomson, "Highly efficient exhaustive search algorithm for optimising canonical Reed-Muller expansions of Boolean functions", *Int. J. of Electronics*, Vol. 76, No. 4, pp. 601-609, 1994.
- [Miller94b] J. F. Miller, P. Thomson, P.V.G. Bradbeer, "The Use of Genetic Algorithms in Reed-Muller Optimisation", *Proc. AISB Workshop in Evolutionary Computing, April 1994*.
- [Miller94c] J.F. Miller, H. Luchian, P.V.G. Bradbeer, P.J. Barclay, "Using a genetic algorithm for optimising fixed polarity Reed-Muller expansions of boolean functions", *Int. J. Electronics*, Vol. 76, No. 4, pp. 601-609, 1994.
- [Miller95] J.F. Miller, P. Thomson and P. V. G. Bradbeer, "Ternary Decision Diagram Optimisation of Reed-Muller Logic Functions using a Genetic Algorithm for Variable and Simplification Rule Ordering", *Lecture Notes in Computer Science Vol 993, "Evolutionary Computing", Springer-Verlag, Berlin. pp. 181-190, 1995*.
- [Miller96] J. F. Miller, P. Thomson, "Restricted Evaluation Genetic Algorithms with Tabu Search for Optimising Boolean Functions as Multi-level AND-EXOR Networks", To appear in *Lecture notes in Computer science "Evolutionary Computing", Springer-Verlag, 1996*.

- [Rechenberg73] I. Rechenberg, "Evolutionsstrategie: Optimierung Technischer Systeme nach Prinzipien der Biologischen Evolution", *Fromann-Helzboog Verlag*, 1973.
- [Reddy72] S.M. Reddy, "Easily testable realisations for logic functions", *IEEE Trans. Comput. Vol. 21*, pp. 1183-1188, 1972.
- [Reeves93] C. Reeves (Ed.) "Modern Heuristic Methods for Combinatorial Problems", *Blackwell Scientific*, 1993.
- [Robertson96] G.I. Robertson, J. F. Miller, P. Thomson, "Non-exhaustive search methods and their use in the minimisation of Reed-Muller canonical expansions", *Int. J. Electronics Vol.80, No. 1*, pp 1-12, 1996.
- [Sasao93] T. Sasao, chapters 12 and 13 in "Logic Synthesis and Optimisation", *Kluwer Academic Publishers*, 1993.
- [Sasao95] T. Sasao, "Representation of Logic Functions using EXOR Operators", *Proc. Workshop on Applications of Reed-Muller Expansion in Circuit Design, Japan*, pp. 11-20, 1995.
- [Thomson96] P. Thomson, J.F. Miller, "Symbolic method for simplifying AND-EXOR representations of Boolean functions using a binary decision technique and a genetic algorithm.", *IEE Proc. Comput. Digit. Tech. Vol. 143, No. 2*, 1996.
- [Zeng95] X. Zeng, M. Perkowski, K. Dill, A. Sarabi, "Approximate Minimisation of Generalised Reed-Muller Forms." *Proc. Workshop on Applications of the Reed-Muller Expansion in Circuit Design*, pp. 221-230, 1995